

2/4/81



INTERNAL LETTER

Date: 2 February 1981

NO: SFR/81-6

TO: Jerry Wroblewski
Loc/Dept: MD-43FROM: Steve Russell
Loc/Dept: MD-45CC: Tom Dennis, MD-43
Jim Marr, MD-43

Phone: 2505

Subject: TCXO Thermistor Resistance Calculations

1. Theory

Several models have been developed to describe the resistance-temperature characteristics of thermistors. The simplest model I know is the Beta Equation. This will give reasonable accuracy over limited temperature ranges.

Beta Equation

$$R(T_n) = R(T_0) e^{\beta_0 \left[\frac{1}{T_n} - \frac{1}{T_0} \right]}$$

ohms

The parameters for the Beta Equation are defined as follows:

T_n = Thermistor body temperature in degrees Kelvin ($25^\circ\text{C} = 298.15^\circ\text{K}$).

T_0 = Temperature at which nominal thermistor resistance, $R(T_0)$, is determined. A reasonable choice is $T_0 = 298.15^\circ\text{K}$ (25°C).

$R(T_0)$ = Thermistor resistance at a body temperature of T_0 .

$R(T_n)$ = Thermistor resistance at a body temperature of T_n .

β_0 = Material constant.

This simple "Beta" model is based on the assumption that the resistance varies inversely with temperature in a "perfect" logarithmic fashion. This can be demonstrated by writing the Beta Equation in its logarithmic form:

$$\ln \left\{ \frac{R(T_n)}{R(T_0)} \right\} = \beta_0 \left[\frac{1}{T_n} - \frac{1}{T_0} \right]$$

When this logarithmic equation is plotted against an actual thermistor curve, the value of β_0 turns out to be slightly temperature dependent. This can be accounted for by trying to "curve fit" β with a polynomial in the Modified Beta Equation.

Modified Beta Equation

If we represent the slight temperature dependence of β by a polynomial, the Beta Equation is modified to give:

$$\beta(T_n) = B_0 + B_1(T_n - T_0) + B_2(T_n - T_0)^2 + B_3(T_n - T_0)^3$$

$$R(T_n) = R(T_0) e^{\beta(T_n) \left[\frac{1}{T_n} - \frac{1}{T_0} \right]}$$

The parameters B_0 , B_1 , B_2 , and B_3 are now material constants to be determined by curve-fitting the logarithmic equation:

$$\beta(T_n) = \left[\frac{1}{T_n} - \frac{1}{T_0} \right]^{-1} \ln \left\{ \frac{R(T_n)}{R(T_0)} \right\}$$

To determine the best least-mean-square error for the B parameters, $\beta(T_n)$ is determined at four spaced temperature values and the following matrix operation performed:

$$\underline{B} = [\underline{T}]^{-1} \underline{D}$$

where:

$$\underline{B} = \begin{bmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} \quad \underline{D} = \begin{bmatrix} \beta(T_1) \\ \beta(T_2) \\ \beta(T_3) \\ \beta(T_4) \end{bmatrix} \quad \underline{T} = \begin{bmatrix} 1 & (T_1 - T_0) & (T_1 - T_0)^2 & (T_1 - T_0)^3 \\ 1 & (T_2 - T_0) & (T_2 - T_0)^2 & (T_2 - T_0)^3 \\ 1 & (T_3 - T_0) & (T_3 - T_0)^2 & (T_3 - T_0)^3 \\ 1 & (T_4 - T_0) & (T_4 - T_0)^2 & (T_4 - T_0)^3 \end{bmatrix}$$

Notice for this model that if β is a constant, then $\beta_0 = B_0$ and the simple Beta model applies.

Western Thermistor Equation

A third model has been developed by Western Thermistor to be used in calculating thermistor values. Their model actually uses four constants that will be designated A, B, C, D. Material constants A, B, and C are determined by experimental measurements. The constant D is determined by the range into which the nominal value falls. In some of their literature, D and R_{25} are used interchangeably which results in some confusion. The logarithmic form of the equation may be written as:

$$\frac{1}{T_n} = A + B \cdot \ln \left\{ \frac{R(T_n)}{R(T_0)} D \right\} + C \cdot \ln^3 \left\{ \frac{R(T_n)}{R(T_0)} D \right\}$$

Internal Letter
February 3, 1981
Page 3

or alternately,

$$\frac{1}{T_n} = A + B \cdot \ln \alpha + C(\ln \alpha)^3$$

with:

$$\alpha = \frac{R(T_n)}{R(T_0)} D$$

If a nominal thermistor value is chosen such that

$$R(T_0) = D = R_{25}$$

then the equation simplifies to the one published in Western Thermistor literature:

$$\frac{1}{T_n} = A + B (\ln R(T_n)) + C(\ln R(T_n))^3$$

The Western thermistor equation is a cubic with no second order term and has a closed form solution. Appendix I shows a simplified form of the solution for thermistor resistance called the Interpolation Equation.

2. Application

For Western Thermistors, each thermistor is characterized by a curve type number. From a Table of R_{25} values, a device is selected and a resulting curve number obtained. Corresponding to a curve number, the parameters A, B, C, and D are determined. The temperature curve or resistance interpolation curve may then be computed.

Example

A 20K ohm thermistor characterized by curve 7 is chosen. From the table of parameters we obtain:

$$\begin{aligned} A &= 9.9790127 \times 10^{-4} \\ B &= 1.9087826 \times 10^{-4} \\ C &= 1.0379321 \times 10^{-8} \\ D &= 100000\Omega \end{aligned}$$

Internal Letter
February 2, 1981
Page 4

Since we have chosen $R(T_0) = 20$ KOHMS, the alpha factor is;

$$\alpha = \frac{R(T_n)}{2 \times 10^4} \cdot 10^5 = 5R(T_n)$$

or conversely

$$R(T_n) = \frac{1}{5} \alpha$$

Using the program in Appendix III, the resistance at three temperatures is calculated by the interpolation equation and compared to the table:

T_n (°C)	$R(T_n)$	$R(T_n)/R(T_0)$	RT/R_{25}
-30	446.7K	22.34	22.35
+25	20.01K	1.001	1.000
+70	2.847K	0.1424	0.1426

Likewise, using the program in Appendix II, the temperature is calculated for three measured resistance values and compared to the table:

$R(T_n)$	T_n (°C) Calc.	T_n (°C) Table
236.0K	-20.00	-20
20K	+25.01	+25
6.420K	+49.99	+40

3. Accuracy of the Model

Tables of thermistor values have been calculated using the equations but their accuracy from batch to batch may vary widely. In addition, there is the nominal tolerance of the device computed at 25°C or 70°C.

Internal Letter
February 2, 1981
Page 5

Variations in actual thermistor performance compared to the interpolation equation can be considerable. For example, a curve 12 thermistor which is normalized to 70°C can have as much as $\pm 13.7\%$ error at -20°C when compared to the value predicted by the equation. Efforts to verify the equations given in Appendix IV have failed. The constant, C, cannot be verified using the values published in the Table. I don't feel it justifies any more time. If the use of these equations becomes important, I'll contact Western and work out the details.

SFR:csf

Appendix I. Simplification of the Western Thermistor Interpolation Equation

The interpolation equation published by Western Thermistor is:

NTC THERMISTOR RESISTANCE VERSUS TEMPERATURE INTERPOLATION EQUATION

$$\ln \frac{R_{T_1}}{R_{T_0}} = \frac{1}{3} \left[\sqrt[3]{ - \frac{27}{2} \left(\frac{A - T_1^{-1}}{C} \right) + \frac{3}{2} \sqrt{ \sqrt[3]{ 27 \left(\frac{A - T_1^{-1}}{C} \right)^2 + 4 \left(\frac{B}{C} \right)^3 } } } - \sqrt[3]{ + \frac{27}{2} \left(\frac{A - T_1^{-1}}{C} \right) + \frac{3}{2} \sqrt{ \sqrt[3]{ 27 \left(\frac{A - T_1^{-1}}{C} \right)^2 + 4 \left(\frac{B}{C} \right)^3 } } } \right]$$

First we note that the Thermistor resistance term should really be α where:

$$\alpha = \frac{R(T_n)}{R(T_0)}^D$$

Using the following substitutions,

$$\theta_1 = \frac{1}{C} \left(A - \frac{1}{T_n} \right)$$

$$\theta_2 = \sqrt[3]{\theta_1^2 + 4 \left(\frac{B}{3C} \right)^3}$$

The interpolation equation is written as:

$$\ln \{ \alpha \} = \frac{1}{\sqrt[3]{2}} \left[\sqrt[3]{ -\theta_1 + \theta_2 } - \sqrt[3]{ +\theta_1 + \theta_2 } \right]$$

Appendix II. HP-25 Program for Calculating Temperature from $R(T_n)$.

				<u>Memory</u>
Enter A, B, C, D, $R(T_o)$, T_o				STO7 A
Enter $R(T_n)$ ohms				STO6 B
01 4 RCL3 ÷ RCL4				STO5 C
16 X + RCL7				STO4 D
05 X f ln STO 1				STO3 $R(T_o)$
20 g 1/x ↑				STO2 273.15°K
RCL2 -				STO1 $\ln a$
10 X 25 GTO00				$\frac{X}{T_n (\text{°C})}$ $\frac{Y}{T_n (\text{°K})}$
X RCL5 X RCL1				
15 RCL6				

Example: (Curve 7)

$$\begin{aligned}
 A &= 9.9790127 \times 10^{-4} \\
 B &= 1.9087826 \times 10^{-4} \\
 C &= 1.0379321 \times 10^{-8} \\
 D &= 100,000 \text{ ohms} \\
 R(T_o) &= 2 \times 10^4 \text{ ohms} \\
 T_o &= 273.15
 \end{aligned}$$

$$R(T_n) = 7.28 \times 10^4$$

$$T_n = 0.073^{\circ}\text{C}$$

Appendix III. HP-25 Program for Thermistor Resistance Calculated from T_n

			Memory
Enter A, B, C, D, $R(T_o)$, T_o			STO7 A
Enter T_n $^{\circ}$ C			STO6 B
01 RCL0			STO5 C
+ 3			STO4 D
g 1/x g 1/x			STO3 $R(T_o)$
CHS f y^x			STO2 θ_2
05 RCL7 30 RCL1			STO1 θ_1
+ RCL2			STO0 273.15 $^{\circ}$ K
RCL5 +			
÷ 3			
STO 1 g 1/x			
10 g x^2 35 f y^x			
RCL6 -			
RCL5 2			
÷ ↑			
3 3			
15 ÷ 40 g 1/x			
↑ f y^x			
↑ ÷			
X ge x^x			$\frac{x}{\alpha}$
X RCL3			
20 4 45 X			
X RCL4			
+ ÷			
f \sqrt{x} 48 GTO00			$\frac{x}{R(T_n)}$ ohms
STO2			
25 RCL 1			

Appendix IV Calculation of Material Constants from Measured Data

Western Thermistor provides the following equations for calculating A, B, and C from measured data:

$$C = \frac{x_3(y_1 - y_2) - x_1(y_1 - y_2) + y_1 x_1 - y_1 x_2 - y_3 x_1 + y_3 x_2}{(y_3)^3(y_1 - y_2) - (y_1)(y_2)^3 + (y_1)^4 - (y_1)^3(y_1 - y_2) + (y_3)(y_2)^3 - (y_3)(y_1)^3}$$

$$B = \frac{x_1 - x_2 + C(y_2)^3 - C(y_1)^3}{(y_1 - y_2)}$$

$$A = x_2 - By_2 - C(y_2)^3$$

where:

$$x_1 = \frac{1}{T_1} \quad y_1 = \ln \alpha_1 \quad \alpha_1 = \frac{R(T_1)}{R(T_0)} D$$

$$x_2 = \frac{1}{T_2} \quad y_2 = \ln \alpha_2 \quad \alpha_2 = \frac{R(T_2)}{R(T_0)} D$$

$$x_3 = \frac{1}{T_3} \quad y_3 = \ln \alpha_3 \quad \alpha_3 = \frac{R(T_3)}{R(T_0)} D$$

All temperatures in $^{\circ}\text{K}$ $0^{\circ}\text{C} = 273.15^{\circ}\text{K}$

D is determined by Curve number

$$-40^{\circ}\text{C} \leq T_1 \leq T_2 \leq T_3 \leq 150^{\circ}\text{C}$$

$$(T_2 - T_1) \leq 50^{\circ}\text{C} \text{ and } (T_3 - T_2) \leq 50^{\circ}\text{C}$$